The answer choice (E) NOTA means that 'none of these answers' are correct. All angles are in radians unless otherwise stated. For all matrices M with nonzero determinant, $M^0 = I$. Good luck, and have fun!

1. Let α be an angle such that $\sin\alpha = \frac{7}{25}$. Evaluate $|\cos\alpha|$.

   (A) $\frac{3}{5}$ (B) $\frac{18}{25}$ (C) $\frac{21}{25}$ (D) $\frac{24}{25}$ (E) NOTA

2. Find the absolute value of z if $z = |3+4i| + 12i$.

   (A) 3 (B) 5 (C) 13 (D) $\sqrt{265}$ (E) NOTA

3. Find $\vec{v}\cdot(\vec{u}\times\vec{w})$ if $\vec{v} = \langle 2,1,2\rangle$, $\vec{u} = \langle -1,4,5\rangle$, and $\vec{w} = \langle -2,-1,0\rangle$.

   (A) 18 (B) -18 (C) $\langle 4,-4,0\rangle$ (D) $\langle 15,-10,9\rangle$ (E) NOTA

4. Find the number of petals on the graph of $r = \sin 3\theta\cos 9\theta + \cos 3\theta\sin 3\theta$.

   (A) 3 (B) 6 (C) 12 (D) 24 (E) NOTA

5. Find the sum of the solutions to $\sin x + \sin 2x + \sin 3x = 0$ where $x \in [0, 2\pi)$.

   (A) 2π (B) 3π (C) 4π (D) 5π (E) NOTA

6. Let $q(x)$ be a quadratic function such that $q(20) = q(18) = 2018$ and $q(22) = q(16) = 2002$. Find $q(21)$.

   (A) 2008 (B) 2010 (C) 2012 (D) 2015 (E) NOTA

For the following three questions, consider the two sets of parametric equations below when graphed in the same plane. They form a single, closed figure.

$$x_1(t) = |\cos(\pi t)| \quad y_1(t) = |\sin(\pi t)|$$
$$x_2(t) = \cos^2(\pi t) \quad y_2(t) = \sin^2(\pi t)$$

7. What is the perimeter of the figure?

   (A) $\frac{\sqrt{2}}{2} + \frac{\pi}{4}$ (B) $\sqrt{2} + \frac{\pi}{4}$ (C) $\frac{\sqrt{2}}{2} + \frac{\pi}{2}$ (D) $\sqrt{2} + \frac{\pi}{2}$ (E) NOTA

8. What is the area contained by the figure?

   (A) $\frac{\pi}{4} - \frac{1}{2}$ (B) $\frac{\pi}{4} - 1$ (C) $\frac{\pi}{2} - \frac{1}{2}$ (D) $\frac{\pi}{2} - 1$ (E) NOTA

9. If t represents time in seconds, how many seconds does it take to draw the figure once, starting at $t = 0$?

   (A) 0.25 (B) 0.5 (C) 1 (D) 2 (E) NOTA

10. Connor and Arnav are writing a math test together (not this one). The probability that one of Arnav's problems can be solved using mass points is 1%. Connor likes mass points a lot more, so the probability that one of his problems can be solved using mass points is 10%. What fraction of the problems should Connor write such that the average probability that a question can be solved using mass points is 4%?

    (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{1}{2}$ (E) NOTA

11. Evaluate $\cos\left(\frac{\pi}{11}\right)+\cos\left(\frac{3\pi}{11}\right)+\cos\left(\frac{5\pi}{11}\right)+\cos\left(\frac{7\pi}{11}\right)+\cos\left(\frac{9\pi}{11}\right)$.

    (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) NOTA

12. In $\triangle ABC$, let D be on $\overline{AB}$ such that $\angle BCD$ is a right angle. If $BC = 3$, $BD = 6$, and $AD = 2$, find AC.

    (A) $3\sqrt{3}$ (B) 6 (C) $4\sqrt{3}$ (D) 7 (E) NOTA

13. Let x and y be real numbers such that $\tan x + \tan y = 5$ and $\cot x + \cot y = 10$. Compute $\tan(x + y)$.

    (A) 5 (B) 10 (C) 15 (D) 20 (E) NOTA

14. Find the volume bounded by the graphs of $x^2 + y^2 = z^2$ and $z^2 - 3z - 18 = 0$ in xyz-space.

    (A) 27π (B) 81π (C) 108π (D) 243π (E) NOTA

15. A function $f(x) = a\sin\theta + b$ exists such that the absolute value of the product of its minimum and maximum values (both integers) is 20. Find the sum of all possible positive values of a.

    (A) 14 (B) 21 (C) 28 (D) 35 (E) NOTA

16. Over a certain domain, (k, ∞), the function $g(x) = \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}$ is equivalent to another function of the form $\frac{a+\sqrt{bx+c}}{d}$, where a and d are relatively prime positive integers. Let k_{min} be the smallest value of k such that this equivalence holds. Find $a+b+c+d+k_{min}$.

    (A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

17. Let M and S_c be matrices such that

$$M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad S_c = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}.$$

    Let A be the sum of all non-negative integral values of K such that there exists a positive integer c such that the sum of the entries of $S_c(M^K)$ is 2018. Let B be the number of such values of K. Find $A + B$.

    (A) 2017 (B) 2018 (C) 3025 (D) 3026 (E) NOTA

18. Let M be a 2018×2018 matrix such each element M_{ij} is given by $M_{ij} = i + 2018(j-1)$. Find the determinant of M.

(A) 2018 (B) 2018^2 (C) 2018! (D) 2018^{2018} (E) NOTA

19. Let the roots of $x^5 + x^4 + x^3 + x^2 + x + 1$ be the vertices of a polygon in the complex plane. Compute the area of this polygon.

(A) $\frac{13\sqrt{3}}{12}$ (B) $\frac{7\sqrt{3}}{6}$ (C) $\frac{5\sqrt{3}}{4}$ (D) $\frac{3\sqrt{3}}{2}$ (E) NOTA

20. Find the equation of the line that bisects the obtuse angle between the lines $3x - 4y = 7$ and $5x + 12y = 9$.

(A) $17x + 3y = 36$ (B) $11x + 4y = 23$ (C) $26x + 5y = 55$
(D) $8x + y = 17$ (E) NOTA

21. Two complex numbers z and w are chosen at random such that $|z| = |w| = 1$. What is the probability that $\text{Re}(zw) > 0$ and $\text{Re}(\frac{z}{w}) > 0$?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) NOTA

22. If $\displaystyle\sum_{k=2}^{\infty} \frac{3k+2}{k^5 + 4k^4 + 5k^3 + 2k^2} = \frac{m}{n}$ where m and n are relatively prime positive integers, then compute the sum $m + n$.

(A) 3 (B) 7 (C) 13 (D) 17 (E) NOTA

23. Evaluate $\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{\pi}{9}\right)\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{4\pi}{9}\right)$.

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) NOTA

24. Evaluate $\displaystyle\lim_{n\to\infty}\left(\frac{1}{n^3} + \frac{4}{n^3} + \frac{9}{n^3} + \frac{16}{n^3} + \frac{25}{n^3} + \ldots + \frac{n^2}{n^3}\right)$.

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) NOTA

25. Let M be a 2017×2017 matrix in which each element M_{ij} is given by

$$M_{ij} = \begin{cases} 1 & \text{if } i + j = 2018 \\ 0 & \text{if } i + j \neq 2018 \end{cases}$$

What is the smallest possible value of $n > 1$ such that $M^n = M$?

(A) 2016 (B) 2017 (C) 2018 (D) 2019 (E) NOTA

26. Evaluate $\displaystyle\prod_{n=1}^{\infty} \cos\left(\frac{x}{2^n}\right)$ at $x = \frac{\pi}{6}$ (Hint: As x approaches 0, $\sin x \approx x$).

(A) $\frac{3}{2\pi}$ (B) $\frac{3}{\pi}$ (C) $\frac{6}{\pi}$ (D) $\frac{9}{\pi}$ (E) NOTA

27. Evaluate $\sum_{k=2}^{2018}\left(k^{\frac{k\pi i}{\ln(k^2)}}\right)$.

(A) -1 (B) $-1-i$ (C) $-i$ (D) 0 (E) NOTA

28. Let θ be a first quadrant angle such that

$$\cos\theta = \frac{\overbrace{\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}}^{\text{2018 2s}}}{2}.$$

If θ can be expressed as $\frac{\pi}{k}$ for positive integer k, evaluate $\lfloor \log_2(k) \rfloor$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. (The 2018 2s does not include the denominator)

(A) 2017 (B) 2018 (C) 2019 (D) 2020 (E) NOTA

29. An infinite amount of boxes with masses 1, 2, 3, ... are arranged in a line. Alex is going to pick up exactly one box; however, he doesn't want to carry anything too heavy, so the probability of him choosing the box with mass m is directly proportional to $\frac{1}{m^3}$. If

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

What is the expected mass of the box he picks up?

(A) 1 (B) $\frac{\zeta(1)}{\zeta(2)}$ (C) $\frac{\zeta(2)}{\zeta(3)}$ (D) $\frac{\zeta(3)}{\zeta(4)}$ (E) NOTA

30. Find the absolute value of

$$\lim_{n\to\infty}\sum_{k=0}^{n-1}\begin{vmatrix} \cos\frac{2\pi k}{n} & \sin\frac{2\pi k}{n} \\ \cos\frac{2\pi(k+1)}{n} & \sin\frac{2\pi(k+1)}{n} \end{vmatrix}$$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) 2π (E) NOTA