The answer choice (E) NOTA denotes that 'none of these answers' are correct. Other necessary acronyms are NEI for 'not enough information' and DNE for 'does not exist.' Good luck, and have fun!

1. Evaluate $\int_0^{\sqrt{3}} \sqrt{x^4+x^2}\,dx$.

   (A) 1 (B) $\frac{4}{3}$ (C) 2 (D) $\frac{7}{3}$ (E) NOTA

2. If $\int_0^{\frac{\pi}{4}} \sec^4(x)\,dx = \frac{a}{b}$ in simplest form, find $a+b$.

   (A) 3 (B) 7 (C) 11 (D) 15 (E) NOTA

3. Let $f(x) = (\ln(x)-2)^4$. If $f^{(4)}(e^2)$ can be written in the form $Ae^B + C$ where A, B, C are rational and B is nonzero, compute $A+B+C$.

   (A) 16 (B) 24 (C) 32 (D) 40 (E) NOTA

4. Compute the largest real number x that is not in the interval of convergence of

$$\sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{x^n}.$$

   (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

5. Given that $\int_0^{\infty} \frac{\cos(x)}{x^2+1}\,dx$ can be written in the form $A\pi^B e^C$ where A, B, C are all rational nonzero numbers, compute ABC.

   (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) -1 (E) NOTA

6. Find the volume when the first quadrant region bounded by the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ is rotated about the x-axis.

   (A) $\frac{8\pi}{105}$ (B) $\frac{16\pi}{105}$ (C) $\frac{32\pi}{105}$ (D) $\frac{64\pi}{105}$ (E) NOTA

7. Evaluate $\lim_{x\to 0} \frac{\sin(x^2) - x^2 - x^6}{e^{x^3} - 1 - x^3}$.

   (A) $-\frac{5}{3}$ (B) $\frac{5}{3}$ (C) $\frac{7}{3}$ (D) $-\frac{7}{3}$ (E) NOTA

8. Find $\sum_{k=0}^{\infty} \frac{1}{(3k)!}$.

   (A) $\ln\frac{3}{2}$ (B) $\ln 2$ (C) $\frac{\sqrt{3}}{3}$ (D) $\frac{4}{3}$ (E) NOTA

9. A certain infinitely many times differentiable function f satisfies

$$f'(x) + xf''(x) = x$$

   Where $f(1) = f'(1) = 0$. Evaluate $f(2)$.

   (A) $\frac{3}{2} - \ln 2$ (B) 1 (C) $\frac{3}{4} - \frac{\ln 2}{2}$ (D) $\frac{3}{2} + \ln 2$ (E) NOTA

10. If the area bound by the curve $y = x^2 - 7x + 2$ and the x-axis can be expressed in the form $\frac{A\sqrt{B}}{C}$ where $\gcd(A, C) = 1$ and B is squarefree, find $A + B + C$.

(A) 82 (B) 84 (C) 86 (D) 88 (E) NOTA

11. Steve runs a seal breeding farm where the number of seals s (in tens) on the farm is given by $\frac{ds}{dt} = k(s)(500 - s)$. Steve, being immortal, notices that after a long amount of time the number of seals on his farm is approcahing a certain number. What is this number?

(A) 250 (B) 500 (C) 750 (D) NEI (E) NOTA

12. Evaluate $\displaystyle\int \cosh(x)\, dx$.

(A) $\cosh(x) + c$ (B) $\sinh(x) + c$ (C) $-\cosh(x) + c$ (D) $-\sinh(x) + c$ (E) NOTA

13. A sequence a_n satisfies the relation $a_{n+2} = 9a_{n+1} - 20a_n$ with $a_1 = 2$ and $a_2 = 9$. Compute

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}.$$

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

14. Let $f(x) = \dfrac{(2-3x)\sqrt{4x^3 + 7x + 5}}{3x^2 + 5}$. If $|f'(1)| = \frac{a}{b}$ in simplest form, find $a + b$.

(A) 115 (B) 123 (C) 155 (D) 181 (E) NOTA

15. A point E exists in rectangle $ABCD$ such that $AE = 7$ and $CE = 24$. $ABCD$ is changing in such a way that AE and CE remain constant, while BE and DE change over time. At one point, $BE = 15$ and BE was growing at a rate of 5 units. At what rate was DE decreasing at that point?

(A) $\frac{10}{3}$ (B) $-\frac{10}{3}$ (C) $\frac{15}{4}$ (D) $-\frac{15}{4}$ (E) NOTA

16. Compute the volume of the figure $9x^2 + 16y^2 - z^2 = 0$ where $0 \le z \le 5$.

(A) $\frac{125\pi}{36}$ (B) $\frac{25}{6}$ (C) $\frac{50\pi}{9}$ (D) $\frac{25\pi}{3}$ (E) NOTA

17. Consider some integrable, infinitely differentiable function $f : \mathbb{R}^+ \to \mathbb{R}^+$. Additionally $f'(x) > 0, f''(x) < 0$ for all x. If A, B, C, D are the exact value $\int_a^b f(x)dx$, the trapezoidal approximation of the integral, the left hand sum approximation, and the right hand approximation respectively then arrange A, B, C, D in order from smallest to largest

(A) $C < A < B < D$ (B) $C < B < A < D$ (C) $D < A < B < C$
(D) $D < B < A < C$ (E) NOTA

18. Evaluate $\displaystyle\frac{d}{dt}\int_0^{x^2} \cos(4t)\, dt$.

(A) $\cos(4x^2)$ (B) $-\sin(4x^2)$ (C) $2x\cos(4x^2)$ (D) $-2x\cos(4x^2)$ (E) NOTA

19. Find $\displaystyle\lim_{x\to 0}\left[x\ln(x^2 + 4x)\right]$

(A) 0 (B) 4 (C) ∞ (D) $-\infty$ (E) NOTA

20. If $\sum_{k=2}^{\infty} \frac{k^2}{k!}$ can be written in the form $A + Be$ for rational a and b, compute $A + B$.

(A) -1 (B) 0 (C) 1 (D) $\frac{3}{2}$ (E) NOTA

21. If $\int_1^2 \frac{dx}{x^7 + x}$ can be written in the form $\frac{A \ln(\frac{B}{C})}{D}$ where A, B, C, D are all positive integers with $\gcd(A, D) = \gcd(B, C) = 1$ and C is as small as possible, find the sum of the digits of $A + B + C + D$.

(A) 16 (B) 17 (C) 18 (D) 19 (E) NOTA

22. Which of the following mathematicians is credited with discovering

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(A) Fermat (B) Riemann (C) Leibniz (D) Newton (E) NOTA

For problems 23-24 refer to the following table:

$f(0) = 4$	$f(1) = 5$	$f'(1) = 4$	$g(0) = 5$	$g(1) = 3$	$g'(1) = 2$

23. Calculate the derivative of $\frac{(f(x))^3 - (g(x))^3}{f(x) - g(x)}$ at $x = 1$.

(A) 50 (B) 62 (C) 74 (D) 86 (E) NOTA

24. Evaluate $\int_0^1 \frac{g'(x) + f'(x)}{g(x) + f(x)}\, dx$.

(A) $\ln \frac{8}{9}$ (B) $\ln \frac{9}{8}$ (C) 1 (D) -1 (E) NOTA

25. Find the area bound by the curve $(x^2 + y^2)^2 = x^3 + xy^2$.

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) 2π (E) NOTA

26. Find the sum of all distinct values of $\frac{dy}{dx}$ for

$$x^3 + 2x^2y + 3yx^2 + y^3 = 7x^2 + 14xy - x^2y + 10$$

where $x = 5$.

(A) -1 (B) 0 (C) 1 (D) 2 (E) NOTA

27. Let $\{x\}$ denote the fractional part of x. Evaluate $15\int_0^{100} \{x\}^2\, dx$.

(A) 600 (B) 800 (C) 1000 (D) 1200 (E) NOTA

28. Buffy has just bought a new house and decides to build a wall to fence in his house. He then, just for fun, decides to build a treehouse to invite the popsicle crew to. His treehouse can be modeled with cross sections of squares taken perpendicular to the x-axis of the curve $y = \ln x$ from $x = 1$ to $x = e$. Assuming the wood he uses is negligibly thin (and anything else that could possibly cause a dispute), find the volume of his tree house.

(A) 1 (B) $e - 1$ (C) $e - 2$ (D) $2e - 4$ (E) NOTA

29. If the abscissa of the x-intercept of the line with negative slope that passes through the point $(2, 3)$ and has minimum length in the first quadrant is k, find $(k - 2)^3$.

(A) 18 (B) $\frac{32}{9}$ (C) $\frac{243}{4}$ (D) 12 (E) NOTA

30. Evaluate

$$\int_0^1 \int_0^{a_1} \int_0^{a_2} \cdots \int_0^{a_{2017}} (a_{2018})\, da_{2018}\, da_{2017}\, da_{2016} \cdots da_1$$

(A) $\frac{1}{2017!}$ (B) $\frac{1}{2018!}$ (C) $\frac{1}{2019!}$ (D) 1 (E) NOTA