

Algebra 1 Individual

AoPS Mu Alpha Theta

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The answer choice **(E)** NOTA denotes that “none of these answers” are correct. Tests are scored such that each correct answer is worth 4 points, each question left blank is worth 0 points, and each incorrect answer is -1 point. All answers must be exact unless otherwise specified, DNE denotes that a function, value, or answer “does not exist”, and $i = \sqrt{-1}$. Good luck, and more importantly, have fun!

- Find the sum of the positive integral factors of 24.
(A) 42 (B) 48 (C) 52 (D) 60 (E) NOTA
- Find the the sum of the coordinates of the midpoint of the point (1009, 1010) and the point (1010, 1009) on the xy -plane.
(A) 2017 (B) 2018 (C) 2019 (D) 2020 (E) NOTA
- If I select a random permutation (arrangement) of the letters in the name "HANNAH", what is the probability it is a palindrome (A *palindrome* is a string of characters that reads the same way forwards and backwards)?
(A) $\frac{1}{720}$ (B) $\frac{1}{240}$ (C) $\frac{1}{120}$ (D) $\frac{1}{90}$ (E) NOTA
- What is the absolute value of the difference of the x - and y -coordinates of the point of intersection of the lines $5x + 3y = 24$ and $y = x$?
(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA
- Rohan takes a number n . He multiplies it by 2, adds 3, divides by 5, and adds 3 again, getting back his original number n . What is n ?
(A) 4 (B) 6 (C) 8 (D) 10 (E) NOTA
- Find the solution that satisfies the equation $\sqrt{x^2 - 4x + 29} = x - 3$.
(A) -10 (B) 3 (C) 5 (D) 10 (E) NOTA
- Vidit and Chhavi are siblings. This year, Vidit's age is $\frac{1}{6}$ times more than Chhavi's age. Ten years ago, Vidit's age was exactly twice Chhavi's age. If the difference between their ages is n years, find n^2 .
(A) 1 (B) 4 (C) 9 (D) 16 (E) NOTA
- Solve for x in the equation $x^{-1} + y^{-1} = 1$, for $x \neq 0$ and $y \neq 0, 1$. Express your answer in terms of y .
(A) y (B) $\frac{y-1}{y}$ (C) $\frac{y}{y-1}$ (D) $\frac{1-y}{y}$ (E) NOTA
- Suppose there is a ladybug that lives at the origin of the xy -plane. The ladybug travels three units north, five units east, eight units south, and seventeen units west from its home. What is the distance from the ladybug's final position to its home, in units?
(A) 17 (B) 13 (C) 10 (D) 5 (E) NOTA
- The line $4x + 3y = 12$ is perpendicular to the line $kx + 5y = 15$. What is k ?
(A) $-\frac{20}{3}$ (B) $\frac{20}{3}$ (C) $\frac{45}{4}$ (D) $-\frac{15}{4}$ (E) NOTA

11. Alex is confused in his algebra class and finds the sum of the roots of the equation, $12x^2 + 7x + 1 = 0$, when he was supposed to find the product of the roots. His friend, Joshua, however, pays attention in class and is easily able to solve the problem correctly. What is the sum of Alex's and Joshua's resulted answers?

(A) $-\frac{1}{2}$ (B) $-\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) NOTA

12. The line passing through points m and n has a slope of 7. Which of the following could be m and n ?

(A) $(-5, 5)$ and $(-16, -82)$ (B) $(28, 4)$ and $(0, 0)$ (C) $(192, 2016)$ and $(199, 2065)$
(D) $(6, 7)$ and $(8, -7)$ (E) NOTA

13. What is the area of the region bounded by the x -axis, y -axis, and the line $7x + 4y = 28$? Express your answer in square units.

(A) 28 (B) 21 (C) 14 (D) 7 (E) NOTA

14. Albert, Brian, and Carolina are trying to write an Algebra 1 test. If Albert takes 10 hours to write an Algebra 1 test, Brian takes 15 hours to write an Algebra 1 test, and Carolina takes 3 hours to write an Algebra 1 test, then how many hours will it take for them to write an Algebra 1 test if all three of them work together?

(A) 28 (B) $\frac{28}{3}$ (C) 3 (D) 2 (E) NOTA

15. The quantity

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 24}}}$$

is equal to a positive integer n . What is n ?

(A) 7 (B) 6 (C) 5 (D) 4 (E) NOTA

16. Which of the following mathematical terms is a proper subset of all the others?

(A) Number (B) Real number
(C) Irrational number (D) Transcendental number
(E) NOTA

17. How many lattice points are on the portion of the line $y = 2x + 4$ that lies above the parabola $y = 15x^2 + 18x - 11$ (A lattice point is a point (x, y) that exists where x and y are both integers)?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

18. The value of x that satisfies $8^{x+6} = 32^{3x+7}$ can be rewritten in the form $\frac{a}{b}$, where a and b are relatively prime integers. Find $|a + b|$.

(A) 5 (B) 6 (C) 7 (D) 8 (E) NOTA

19. Let t_n be an arithmetic sequence such that $t_n = 2n - 1$. What is the sum of the first 75 terms of this sequence?

(A) 5400 (B) 5551 (C) 5625 (D) 5775 (E) NOTA

20. Given $a = 4$, $b = 2$, and $c = 3$, find the value of the expression

$$\frac{(ab^2c^3)^2 \cdot ab^4c^3}{(abc)^6}.$$

- (A) $\frac{27}{16}$ (B) $\frac{3}{4}$ (C) $\frac{81}{32}$ (D) $\frac{9}{8}$ (E) NOTA
21. Find the solution set of the inequality $\frac{1}{4} \leq \frac{2}{1-x}$.
- (A) $(\infty, -7] \cup (1, \infty)$ (B) $(\infty, -7) \cup (1, \infty)$ (C) $(-7, 1)$
(D) $[-7, 1)$ (E) NOTA
22. Tanvi walks up an escalator at a constant speed of 3.2 km/h (the escalator is going downward). After reaching the top of the escalator, she goes down the same escalator at a constant speed of 9.6 km/h. What is the speed of the escalator that Tanvi walked on? Express your answer in meters per second. (Note: There are 1000 meters in a kilometer.)
- (A) $\frac{16}{5}$ (B) $\frac{32}{5}$ (C) $\frac{8}{9}$ (D) $\frac{16}{9}$ (E) NOTA
23. The sum of the roots of my favorite quadratic is a number k . The product of the roots is $k + 4$ and the leading coefficient of my favorite quadratic is 1. Also, my favorite quadratic can be expressed as $(x - n)^2$ for some value of n . Which of the following is a possible value of k ?
- (A) -6 (B) 0 (C) 2 (D) $2 + 2\sqrt{5}$ (E) NOTA
24. Find the sum of the last two digits of $5^{100} + 6^{67} + 7^{98}$.
- (A) 1 (B) 3 (C) 5 (D) 7 (E) NOTA
25. Find the positive value of x satisfying $\frac{x+4}{x+2} + \frac{x+2}{x+4} = \frac{25}{12}$.
- (A) 4 (B) 5 (C) 7 (D) 10 (E) NOTA
26. The unique value of c that makes the following system have exactly one solution can be expressed in simplest form as $\frac{p}{q}$. Find $p + q$.
- $$\begin{aligned} 20x + 19y &= 18 \\ y &= |2x - c| \end{aligned}$$
- (A) 11 (B) 12 (C) 13 (D) 14 (E) NOTA
27. The polynomial $x^3 + x^2 - 4x - 4$ can be factored as $(x - a)(x - b)(x - c)$, where a , b , and c are integers. What is $|a| + |b| + |c|$?
- (A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA

28. How many of the following numbers are irrational?

i. e

iii. π

v. $\sqrt{1681}$

ii. the positive solution to $2x^2 - 5x - 7 = 0$

iv. $\sqrt{2}$

vi. ϕ (the Golden Ratio, $\frac{1+\sqrt{5}}{2}$)

(A) 3

(B) 4

(C) 5

(D) 6

(E) NOTA

29. Jaehyun, the expert chemist, decides to mix two solutions together. The first solution is 30 milliliters and is made of 30% acid. The second solution is x milliliters of pure acid. Once mixed, these solutions create another solution that is 44% acid. What is the value of x ?

(A) 6

(B) 7

(C) 8

(D) 9

(E) NOTA

30. It takes four days for three workers to paint one house in my town. Assume all workers work at the same rate and that all houses take an equal amount of time to paint. Suppose I call over ten workers to paint my house. After one day, half the workers are exhausted and go home. I decide to pay each remaining worker a \$10 tip for each additional whole hour it takes to finish the job (after the other workers leave). How many dollars do I end up paying all the remaining workers for these tips?

(A) 450

(B) 500

(C) 650

(D) 700

(E) NOTA