Remember, the choice NOTA means "None of the Aforementioned". For all questions, all numbers are assumed to be real unless indicated and $i = \sqrt{-1}$. Good luck and have fun!

1. Compute $\dfrac{4 + 5 \cdot 6 + 91 - 285 + 2018}{2}$.

   (A) 907 (B) 913 (C) 926 (D) 929 (E) NOTA

2. What is the degree of the monomial $x^7y^3z^9(x^5y^7z^3)^2$?

   (A) 30 (B) 36 (C) 46 (D) 49 (E) NOTA

3. Simplify $\dfrac{6}{5+i}$.

   (A) $\frac{15-3i}{13}$ (B) $\frac{5-i}{4}$ (C) $\frac{30-6i}{23}$ (D) $\frac{30-i}{24}$ (E) NOTA

4. There exists a function $f(x)$. What is the real value of $f^{-1}(4099)$, if $f(x) = 8x^3 + 3$?

   (A) -8 (B) 8 (C) 16 (D) 64 (E) NOTA

5. Ritam bikes to school at an average speed of 25 mph on Monday, making him 5 minutes early. He bikes to school at an average speed of 20 mph on Tuesday, making him 5 minutes late. At what average speed, in miles per hour, must Ritam bike on Wednesday in order to get to school on time, given he travels the same distance each day? Express your answer as a decimal to the nearest tenth.

   (A) 22.2 (B) 22.3 (C) 22.4 (D) 22.5 (E) NOTA

6. Given that $\ln 15 = x$, what is e^{x+5}?

   (A) e^{20} (B) $15e^5$ (C) 75 (D) 759375 (E) NOTA

7. Solve the inequality $\frac{x^2-8x+12}{x-6} \geq 0$. Express your answer in interval notation.

   (A) $[2, \infty)$ (B) $(-\infty, 2]$ (C) $[2, 6) \cup (6, \infty)$
   (D) $(2, 6) \cup (6, \infty)$ (E) NOTA

8. Let $f(x) = 7x + 16$. What is the sum of the possible values of x that satisfy the equation $[f(x)]^{-1} = f^{-1}(x)$?

   (A) $\frac{24}{7}$ (B) $\frac{48}{7}$ (C) -8 (D) $-\frac{96}{7}$ (E) NOTA

9. x varies directly with y^5 and inversely with $\sqrt[3]{z}$. If $y = 2$ and $z = 27$ when x is 11, then what is x when $y = 3$ and $z = 8$?

   (A) $\frac{22}{3}$ (B) $\frac{8019}{64}$ (C) $\frac{1317}{32}$ (D) $\frac{11}{6}$ (E) NOTA

10. Solve:

$$\begin{vmatrix} 25 & 75 \\ -4 & -5 \end{vmatrix}$$

    (A) 1855 (B) 275 (C) 175 (D) -425 (E) NOTA

11. What is the distance between $5 + 10i$ and $-15 + 70i$ in the complex plane?

(A) $10i\sqrt{37}$ (B) $20i\sqrt{17}$ (C) $20\sqrt{10}$ (D) $10\sqrt{65}$ (E) NOTA

12. Which of the following equations, when graphed, are invertible?
i. $f(x) = 2x + 4$ ii. $f(x) = x^2 - 4x + 3$
iii. $f(x) = e^x$ iv. $x = e^\pi$

(A) i, iii, and iv (B) i only (C) i and iii only (D) i, ii, and iii (E) NOTA

13. Let $\log 2 = a$, $\log 5 = b$, and $\log 7 = c$. What is $10^{4a+2b+3c}$?

(A) 3900 (B) 4200 (C) 7800 (D) 8400 (E) NOTA

14. How many terms are in the expansion of $(2x - 3y + 5z)^5$?

(A) 35 (B) 28 (C) 21 (D) 14 (E) NOTA

15. Find how many positive integer triples (a, b, c) satisfy the equation:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

(A) 3 (B) 6 (C) 10 (D) 15 (E) NOTA

16. How many of the following functions are rational?

i. $f(x) = 1$ ii. $f(x) = x$ iii. $f(x) = \dfrac{2^x}{x}$
iv. $f(x) = \dfrac{\log(x)}{x - 1}$ v. $f(x) = \dfrac{x^2 - 5x + 7}{7x - 9}$ vi. $f(x) = \dfrac{x^{0.5}}{2x - 1}$

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

17. Find the sum of the reciprocals of the roots of the fifth-degree polynomial, $95x^5 - 96x^4 + 97x^3 - 98x^2 + 99x - 100$.

(A) $\frac{19}{20}$ (B) $\frac{97}{99}$ (C) $\frac{95}{96}$ (D) $\frac{99}{100}$ (E) NOTA

18. Find the remainder when $x^{99} + 1$ is divided by $x^2 - 2x + 1$.

(A) $99x - 97$ (B) $99x - 98$ (C) $1 - x$ (D) 0 (E) NOTA

19. Evaluate $x + y$, if x is the sum of the digits and y is the number of digits in the number resulted from the operation:

$$\prod_{n=1}^{10} n \cdot \sum_{n=1}^{10} 2n - 1$$

(A) 20 (B) 32 (C) 36 (D) 44 (E) NOTA

20. What is the shortest distance, in units, between the point $(3, -4, 12)$ and the plane defined by the equation $10x - 6y - 15z = -49$?

(A) 4 (B) $\frac{77}{19}$ (C) $\frac{77}{13}$ (D) 6 (E) NOTA

21. What is the greatest integer less than or equal to

$$\ln(2018^{\frac{2018}{\ln 2018}})?$$

(A) 2016 (B) 2017 (C) 2018 (D) 2019 (E) NOTA

22. What is the term below that is defined by the equation $25x^2 + 4y^2 + 200x - 32y = -64$?

(A) Hyperbola (B) Parabola (C) Ellipse (D) Circle (E) NOTA

23. What is the equation of the directrix of the graph of $y = \frac{1}{4}x^2 - 3x + 25$?

(A) $y = 15$ (B) $y = 12$ (C) $x = 15$ (D) $x = 12$ (E) NOTA

For questions 24 and 25, let M represent a non-degenerate conic section with the equation $16x^2 - 9y^2 + 160x - 90y = -751$.

24. What is the distance, in units, between the foci of the graph of M?

(A) 5 (B) 10 (C) 16 (D) 20 (E) NOTA

25. What is the eccentricity of the graph of M?

(A) 0 (B) $\frac{\sqrt{7}}{4}$ (C) 1 (D) $\frac{5}{4}$ (E) NOTA

26. Define the *product root* of a number to be a calculation done by simply taking the product of its digits until it is a one-digit number. For example, the product root of 2117 is 4, as $2 \cdot 1 \cdot 1 \cdot 7 = 14$ and $1 \cdot 4 = 4$. Let $p(n)$ represent the product root of a number n.
Find the value of:

$$\sum_{n=1}^{50} p(n)$$

(A) 177 (B) 225 (C) 330 (D) 351 (E) NOTA

27. If $x + \frac{1}{x} = 3$, then $x^7 + \frac{1}{x^7} = K$ for an integer K. Compute the sum of the digits of K.

(A) 14 (B) 15 (C) 16 (D) 17 (E) NOTA

28. Let α, β, and μ be the roots of $x^3 - 5x^2 + 3x + 7$. If $(\alpha^2 + 1)(\beta^2 + 1)(\mu^2 + 1)$ can be written in the form $a + bi$ for rational numbers a and b, then compute $a + b$.

(A) 104 (B) 128 (C) 152 (D) 176 (E) NOTA

29. If the sum of the magnitudes of the solutions to the equation $(x+1)(x+2)(x+3)(x+4) = 8$ can be written in the form $\sqrt{A} + \sqrt{B}$ where A and B are positive integers, compute $A + B$.

(A) 49 (B) 65 (C) 84 (D) 101 (E) NOTA

30. Evaluate:

$$\frac{1}{\log_5 49} \cdot \frac{1}{\log_6 3125} \div \frac{\log_7 216}{2}$$

(A) $\frac{4\log^2 7}{3\log^2 6}$ (B) $\frac{7}{30}$ (C) $\frac{1}{5}$ (D) $\frac{1}{15}$ (E) NOTA